# **Determination of the equation of state of quark matter from** *J/ψ* **and** *Υ* **suppression at RHIC and LHC**

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**Abstract.** The long lifetime of the quark–gluon plasma likely to be created in relativistic heavy ion collisions at RHIC and LHC energies renders it sensitive to the details of the equation of state of the quark matter. We show that the  $p_T$  dependence of the survival probability of the directly produced  $J/\psi$  at RHIC energies and that of the directly produced  $\Upsilon$  at LHC energies is quite sensitive to the speed of sound in the quark matter, which relates the pressure and the energy density of the plasma. The transverse expansion of the plasma is shown to strongly affect the  $J/\psi$  suppression at LHC energies.

## **1 Introduction**

Statistical quantum chromodynamics predicts that at sufficiently high densities or temperatures the quarks and gluons confined inside hadrons undergo a deconfining phase transition to a plasma of quarks and gluons. The last two decades of high energy nuclear physics activity has been directed towards the production of this new state of matter through relativistic heavy ion collisions. This has led to experiments at BNL AGS and CERN SPS and to the building of the BNL Relativistic Heavy Ion Collider and the planning of the ALICE experiment at the CERN Large Hadron Collider. With the reported confirmations of the quark–hadron phase transition at the relativistic heavy ion collision experiments at the CERN SPS [1], the first step in the search for quark–gluon plasma, which pervaded the early universe, microseconds after the big bang and which may be present in the core of neutron stars, is complete.

The emphasis of the experiments at the BNL RHIC and the CERN LHC will now necessarily shift to an accurate determination of the properties of the quark matter. An important observable for this is the speed of sound in the plasma, defined through

$$
c_{\rm s}^2 = \frac{\partial p}{\partial \epsilon}.\tag{1}
$$

Often one writes

$$
p = c_s^2 \epsilon \tag{2}
$$

for the equation of state of the quark matter, where  $\epsilon$  is the energy density and  $p$  is the pressure. For the simplest bag-model equation of state (with  $\mu_B = 0$ ), we write

$$
\epsilon = 3aT^4 + B,\tag{3}
$$

$$
p = aT^4 - B,\t\t(4)
$$

$$
a = \left[2 \times 8 + \frac{7}{8} \times 2 \times 2 \times 3 \times N_f\right] \frac{\pi^2}{90},\tag{5}
$$

with B as the bag pressure and  $N_f$  as the number of flavors, so that  $c_s^2 = 1/3$ . In general  $\Delta = \epsilon - 3p$  measures the deviation of the equation of state from the ideal gas of massless quarks and gluons (when it is identically zero) and depends sensitively on the interactions present in the plasma. The lattice QCD calculations show that  $\Delta \geq 0$ , until the temperature is several times the critical temperature [2]. This implies that in general  $c_s^2 \leq 1/3$ . Any experimental information on this will be most welcome.

We show in the present work that the transverse momentum dependence of the survival probability of the  $J/\psi$ and  $\Upsilon$  at RHIC and LHC energies are quite sensitive to the value of the speed of sound. The very long lifetime of the plasma likely to be attained at LHC makes it even more sensitive to the details of the equation of state of the quark matter through the transverse expansion of the plasma.

# **2 Formulation**

The theory of quarkonium suppression in QGP [3] is very well studied and several excellent reviews exist [4–6], which dwell both on the phenomenology as well as on the experimental situation. We recall the basic details which are relevant for the present demonstration.

The interquark potential for (non-relativistic) quarkonium states at zero temperature may be written as

$$
V(r,0) = \sigma r - \frac{\alpha}{r},\tag{6}
$$

**Table 1.** Critical screening masses et cetera for quarkonia [7, 8]

State	$J/\psi$	$\chi_c$	$\psi'$	Υ		$\chi_b$
$M$ (GeV)	3.1	3.5	3.7	9.4	10.0	9.9
$r$ (fm)	0.45	0.70	0.88	0.23	0.51	0.41
$\tau_F$ (fm)	0.89	2.0	1.5	0.76	1.9	2.6
$\mu_{\rm D}$ (GeV)	0.70	0.34	0.36	1.57	0.67	0.56
$T_{\rm d}/T_{\rm c}$	1.17	1.0	1.0	2.62	1.12	1.0
$\epsilon_{\rm s}$ (GeV/fm <sup>3</sup> )	1.92	1.12	1.12	43.37	1.65	1.12

where r is the separation between Q and  $\overline{Q}$ . The bound states of  $c\bar{c}$  and  $b\bar{b}$  are well described if the parameters  $\sigma = 0.192 \,\text{GeV}^2$ ,  $\alpha = 0.471$ ,  $m_c = 1.32 \,\text{GeV}$ , and  $m_b =$ 4.746 GeV are used [7]. At finite temperatures the potential is modified due to colour screening, and evolves to

$$
V(r,T) = \frac{\sigma}{\mu(T)} \left[ 1 - e^{-\mu(T)r} \right] - \frac{\alpha}{r} e^{-\mu(T)r}.
$$
 (7)

The screening mass increases with temperature. When  $\mu(T) \rightarrow 0$ , (6) is recovered. At finite temperature, when  $r \to 0$  the  $1/r$  behavior is dominant, while as  $r \to \infty$  the range of the potential decreases with  $\mu(T)$ . This makes the binding less effective at finite temperature. Semiclassically, one can write for the energy of the pair

$$
E(r,T) = 2m_Q + \frac{c}{m_Q r^2} + V(r,T),
$$
 (8)

where  $\langle p^2 \rangle \langle r^2 \rangle = c = \mathcal{O}(1)$ . The radius of the bound state at any temperature is obtained by minimizing  $E(r, T)$ . Beyond some critical value  $\mu_D$  for the screening mass  $\mu(T)$ , no minimum is found. The screening is nowstrong enough to make the binding impossible and the resonance cannot form in the plasma. The ground state properties of some of the quarkonia reported by authors of [7] are given in Table 1. We have also listed the formation time of these resonances defined in [8] as the time taken by the heavy quark to traverse a distance equal to the radius of the quarkonium in its rest frame  $\sim m_Q r_{Q\overline{Q}}/p_{Q\overline{Q}}$ , where  $p_{Q\overline{Q}}$ is the momentum of either of the quarks of the resonance. It may be recalled that somewhat different values for the formation time are reported by Blaizot and Ollitrault [9], who solve the bound state problem within the WKB approximation and define the formation time as the time spent by a quark in going between the two classical turning points.

Nowlet us consider a central collision in a nucleus– nucleus collision, which results in the formation of quark– gluon plasma at some time  $\tau_0$ . Let us concentrate on  $z = 0$ and on the region of the energy density  $\epsilon \geq \epsilon_{s}$  which encloses the plasma which is dense enough to cause the melting of a particular state of quarkonium. We assume the plasma to cool, according to Bjorken's boost invariant (longitudinal) hydrodynamics and then generalize our results to include the transverse expansion of the plasma. We assume that the  $Q\overline{Q}$  pair is produced at the transverse position **r** at  $\tau = 0$  on the  $z = 0$  plane with momentum  $\mathbf{p}_T$ . In the collision frame, the pair would take a time equal to  $\tau_F E_T/M$  for the quarkonium to form, where  $E_T = (p_T^2 + M^2)^{1/2}$  and M is the mass of the quarkonium. During this time, the pair would have moved to the location  $(\mathbf{r}+\tau_F\mathbf{p}_T/M)$ . If at this instant, the plasma has cooled to an energy density less than  $\epsilon_s$ , the pair would escape and quarkonium would be formed. If however, the energy density is still larger than  $\epsilon_s$ , the resonance will not form and we shall have a quarkonium suppression [9–11].

It is easy to see that the  $p_T$  dependence of the survival probability will depend on how rapidly the plasma cools. If the initial energy density is sufficiently high, the plasma will take longer to cool and only the pairs with very high  $p<sub>T</sub>$  will escape. If however the plasma cools rapidly, then even pairs with moderate  $p<sub>T</sub>$  will escape. The transverse expansion of the plasma can further accelerate the rate of cooling giving us an additional handle to explore the equation of state, which as we know, will control the expansion of the plasma.

#### **2.1 Longitudinal expansion of the plasma**

As indicated, we first take the Bjorken boost invariant longitudinal hydrodynamics to explore the expansion of the plasma. Thus, the energy-momentum tensor of the plasma is written as [12]

$$
T^{\mu\nu} = (\epsilon + p)u^{\mu}u^{\nu} + g^{\mu\nu}p,\tag{9}
$$

where  $\epsilon$  is the energy density, p is the pressure, and  $u^{\mu}$  is the four-velocity of the fluid, in a standard notation. If the effects of viscosity are neglected, the energy-momentum conservation is given by

$$
\partial_{\mu}T^{\mu\nu} = 0. \tag{10}
$$

The assumption of the boost invariance implies that the energy density, pressure, and temperature become functions of only the proper time  $\tau$  and that (10) simplifies to

$$
\frac{\mathrm{d}\epsilon}{\mathrm{d}\tau} = -\frac{\epsilon + p}{\tau}.\tag{11}
$$

The effect of the speed of sound is seen immediately. Using (2), we can now write

$$
\epsilon(\tau)\tau^{1+c_s^2} = \epsilon(\tau_0)\tau_0^{1+c_s^2} = \text{const.}\tag{12}
$$

so that if  $c_s^2$  is small, the cooling is slower. Chu and Matsui [10] explored the consequence of the extrema  $c_s^2 = 1/3$ and  $c_s^2 = 0$  on the  $p_T$  dependence of the survival probability. We shall explore the sensitivity of the quarkonium suppression to the equation of state by (somewhat arbitrarily) choosing two values of the speed of sound,  $1/3^{1/2}$ and  $1/5^{1/2}$ , in the following.

We now have all the ingredients to write down the survival probability and we closely follow Chu and Matsui for this.

We take a simple parametrization for the energy density profile:

$$
\epsilon(\tau_0, r) = \epsilon_0 \left[ 1 - \frac{r^2}{R^2} \right]^{\beta} \theta(R - r) \tag{13}
$$

where  $r$  is the transverse coordinate and  $R$  is the radius of the nucleus. One can define an average energy density  $\langle \epsilon_0 \rangle$  by

$$
\pi R^2 \langle \epsilon_0 \rangle = \int 2\pi r \mathrm{d}r \epsilon(r), \qquad (14)
$$

so that

$$
\epsilon_0 = (1+\beta)\langle \epsilon_0 \rangle. \tag{15}
$$

We have taken  $\beta = 1/2$ , which may be thought of as indicative of the energy deposited being proportional to the number of participants in the system. In case one feels that the energy deposited may be proportional to number of nucleon–nucleon collisions then one can repeat the calculations with  $\beta = 1$ , which will reflect the proportionality of the deposited energy to the nuclear thickness. The average energy density is obtained from the Bjorken formula:

$$
\langle \epsilon_0 \rangle = \frac{1}{\pi R_{\rm T}^2 \tau_0} \frac{\mathrm{d} E_{\rm T}}{\mathrm{d} y} \tag{16}
$$

where  $E_T$  is the transverse energy deposited in the collision.

The time  $\tau_s$  when the energy density drops to  $\epsilon_s$  is easily estimated as

$$
\tau_{\rm s}(r) = \tau_0 \left[ \frac{\epsilon(\tau_0, r)}{\epsilon_{\rm s}} \right]^{1/(1+c_{\rm s}^2)}
$$

$$
= \tau_0 \left[ \frac{\epsilon_0}{\epsilon_{\rm s}} \right]^{1/(1+c_{\rm s}^2)} \left[ 1 - \frac{r^2}{R^2} \right]^{\beta/(1+c_{\rm s}^2)} \tag{17}
$$

As discussed earlier [10], we can equate the duration of screening  $\tau_{\rm s}(r)$  to the formation time  $t_{\rm F} = \gamma \tau_{\rm F}$  for the quarkonium to get the critical radius,  $r_s$ :

$$
r_{\rm s} = R \left[ 1 - \left( \frac{\gamma \tau_{\rm F}}{\tau_{\rm s0}} \right)^{(1 + c_{\rm s}^2)/\beta} \right]^{1/2} \theta \left[ 1 - \frac{\gamma \tau_{\rm F}}{\tau_{\rm s0}} \right],\qquad(18)
$$

where  $\tau_{s0} = \tau_s(r=0)$ . This critical radius is seen to mark the boundary of the region where the quarkonium formation is suppressed. As discussed earlier, the quark pair will escape the screening region (and form quarkonium) if its position and transverse momentum  $p_T$  are such that

$$
|\mathbf{r} + \tau_{\rm F} \mathbf{p}_{\rm T}/M| \ge r_{\rm s}.\tag{19}
$$

Thus, if  $\phi$  is the angle between the vectors **r** and  $\mathbf{p}_T$ , then

$$
\cos \phi \ge \left[ (r_s^2 - r^2) M - \tau_F^2 p_\text{T}^2 / M \right] / \left[ 2r \tau_\text{F} p_\text{T} \right],\tag{20}
$$

which leads to a range of values of  $\phi$  when the quarkonium would escape. We also realize that if the right hand side of the above equation is greater than 1, then no angle is possible when the quarkonium can escape. Now we can write for the survival probability of the quarkonium

$$
S(p_{\rm T}) = \left[ \int_0^R r \mathrm{d}r \int_{-\phi_{\rm max}}^{+\phi_{\rm max}} \mathrm{d}\phi P(\mathbf{r}, \mathbf{p}_{\rm T}) \right]
$$

$$
/ \left[ 2\pi \int_0^R r \mathrm{d}r P(\mathbf{r}, \mathbf{p}_{\rm T}) \right], \tag{21}
$$

where  $\phi_{\text{max}}$  is the maximum positive angle  $(0 \leq \phi \leq \pi)$ allowed by (20), and

$$
\phi_{\max} = \begin{cases} \pi & \text{if } y \le -1, \\ \cos^{-1} |y| & \text{if } -1 < y < 1, \\ 0 & \text{if } y \ge 1, \end{cases}
$$
 (22)

where

$$
y = \left[ (r_s^2 - r^2)M - \tau_{\rm F}^2 p_{\rm T}^2 / M \right] / \left[ 2r \tau_{\rm F} p_{\rm T} \right],\qquad(23)
$$

and  $P$  is the probability for the quark-pair production at **r** with transverse momentum  $p_T$ , in a hard collision. Assuming that the  $p_T$  and **r** dependence for hard collisions factor out, we approximate

$$
P(\mathbf{r}, \mathbf{p}_T) = P(r, p_T) = f(r)g(p_T), \tag{24}
$$

where we take

$$
f(r) \propto \left[1 - \frac{r^2}{R^2}\right]^\alpha \theta(R - r),\tag{25}
$$

with  $\alpha = 1/2$ . Equation (21) can be solved analytically for some limiting cases of  $p_T$  et cetera; see [10].

#### **3 Transverse expansion of the plasma**

It is generally accepted that the rarefaction wave from the surface of the plasma will reach the center by  $\tau = R/c_s$ . For the case of lead nuclei, this amounts to about  $12 \, \text{fm}/c$ . If the lifetime of the QGP is comparable to this time, the transverse expansion of the plasma cannot be ignored. The transverse expansion of the plasma will lead to a much more rapid cooling than suggested by a purely longitudinal expansion.

For the four-velocity of the collective flowwe write

$$
u^{\mu} = (\gamma, \gamma \mathbf{v}),\tag{26}
$$

where **v** is the collective flow velocity and  $\gamma = 1/(1 (v^2)^{1/2}$ . We further assume that the longitudinal flow of the plasma has a scaling solution, so that the boost invariance along the longitudinal direction remains valid. Assuming cylindrical symmetry, valid for central collisions, it can be shown that the four-velocity  $u^{\mu}$  should have the form

 $u^{\mu} = \gamma_r(\tau, r)(t/\tau, v_r \cos \phi, v_r \sin \phi, z/\tau),$  (27)

with

$$
\gamma_r = \left[1 - v_r^2(\tau, r)\right]^{-1/2}, \n\tau = (t^2 - z^2)^{1/2}, \n\eta = \frac{1}{2} \ln \frac{t+z}{t-z}.
$$
\n(28)

Thus all the Lorentz scalars are now functions of  $\tau$  and r, and independent of the space-time rapidity  $\eta$ . This reduces the  $(3 + 1)$  dimensional expansion with cylindrical symmetry and boost invariance along the longitudinal direction to

$$
\partial_{\tau} T^{00} + r^{-1} \partial_{r} (r T^{01}) + \tau^{-1} (T^{00} + p) = 0 \tag{29}
$$

and

$$
\partial_{\tau}T^{01} + r^{-1}\partial_{r}\left[r(T^{00} + p)v_{r}^{2}\right] + \tau^{-1}T^{01} + \partial_{r}p = 0, (30)
$$

where

$$
T^{00} = (\epsilon + p)u^0u^0 - p \tag{31}
$$

and

$$
T^{01} = (\epsilon + p)u^0u^1.
$$
\n
$$
(32)
$$

We see that the speed of sound which will appear through the dependence of the pressure p on the energy density will affect the time evolution of all the quantities, especially through the gradient terms. This is of course extensively documented. We solve these equations using well established methods [13] with initial energy density profiles as before and estimate the constant energy density contours [14, 15] appropriate for  $\epsilon = \epsilon_s$  to get  $\tau_s(r)$ . The rest of the treatment follows as before. In these calculations we have assumed the initial transverse velocity to be identically zero.

We only need to identify the initial conditions. We consider  $Pb + Pb$  collisions  $(Au + Au, for RHIC)$  with the initial average energy densities

$$
\langle \epsilon_0 \rangle = \begin{cases} 6.3 \,\text{GeV} / \text{fm}^3, \text{ SPS}, \, \tau_0 = 0.5 \,\text{fm}, \\ 60 \,\text{GeV} / \text{fm}^3, \text{ RHIC}, \, \tau_0 = 0.25 \,\text{fm}, \\ 425 \,\text{GeV} / \text{fm}^3, \text{ LHC}, \, \tau_0 = 0.25 \,\text{fm}. \end{cases} \tag{33}
$$

The estimate for SPS is obtained from the assumption of  $QGP$  formation in  $Pb + Pb$  experiments, while those for RHIC and LHC are taken from the self-screened parton cascade calculation [16]. If the formation time for the SPS energies is assumed to be of the order of  $1 \text{ fm}/c$ , the estimate given above will drop to about  $3 \text{ GeV}/\text{fm}^3$ . A larger initial energy density than the one assumed here for RHIC and LHC could be obtained by using the concept of parton saturation [17]. We are, however, interested only in a demonstration of the effect of the equation of state on the quarkonium suppression, and we feel that the values used here are enough for this.

## **4 Results**

#### **4.1 Speed of sound versus transverse expansion**

It is quite clear that a competition between the speed of sound and the onset of the transverse expansion during the lifetime of the deconfining matter can lead to interesting possibilities. In order to illustrate this and to explore the consequences, we show in Fig. 1 the time corresponding to the constant energy density contours which enclose the deconfining matter – which can dissociate the directly produced  $J/\psi$  (see Table 1), at RHIC energies. We see that owing to the (relatively) short time that the QGP would



**Fig. 1.** The contour of deconfining matter capable of suppressing direct  $J/\psi$  formation at RHIC energies. The solid curves are results with transverse flow, while the dashed curves are for longitudinal flow



**Fig. 2.** The contour of deconfining matter capable of suppressing direct  $J/\psi$  formation at LHC energies. The solid curves are results with transverse flow, while the dashed curves are for longitudinal flow

take to cool down to  $\epsilon_s^{J/\psi}$ , the effect of the transverse flow is marginal and for  $c_s^2 = 1/3$  limited to large radii. Large changes in the contour are seen when the speed of sound is varied. This is very important indeed. Note that the cooling to the value appropriate for  $\gamma$  suppression is attained too quickly to be affected by the transverse expansion, and even the change due to variation in the speed of sound is quite small. Of course the duration of the deconfining medium is prolonged if the speed of sound is reduced.

The corresponding results for the LHC energies are shown in Fig. 2. Now we see that at  $r = 0$  the duration of the deconfining medium reduces by a factor of 2 when the speed of sound is  $1/3^{1/2}$ , and the transverse expansion of



**Fig. 3.** The contour of deconfining matter capable of suppressing direct  $\Upsilon$  formation at LHC energies. The solid curves are results with transverse flow, while the dashed curves are for longitudinal flow



**Fig. 4.** The survival probability for  $J/\psi$  at SPS, RHIC, and LHC energies, with only longitudinal cooling, when the speed of sound is changed

the plasma is allowed. This is a consequence of the longer time which the plasma takes to cool at LHC.

The scenario for the  $\Upsilon$  dissociating matter at LHC is quite akin to the case of  $J/\psi$  dissociating medium at RHIC; the results are affected by the speed of sound and not by the transverse flow (Fig. 3).

#### **4.2 Consequences for survival of quarkonia**

Nowwe return to the transverse momentum dependence of the survival probabilities. As a first step, we plot the survival of the directly produced  $J/\psi$ 's at SPS, RHIC and LHC energies (Fig. 4) when only longitudinal expansion is accounted for and the speed of sound is varied. We see that



**Fig. 5.** The survival probability for  $J/\psi$  at RHIC energy with longitudinal and transverse cooling when the speed of sound is changed

RHIC energies provide the most suitable environment to measure the speed of sound with the help of  $J/\psi$  suppression. The variations in the  $p_T$  dependence are too small at SPS (due to a very short duration of the deconfining medium) and at LHC (now, due to a very long duration!) when the speed of sound is varied.

This advantage of RHIC energies is maintained when the transverse expansion is accounted for (Fig. 5). As one could have expected from the contours (Fig. 1), the results are more sensitive to the variation of the speed of sound than to the transverse flow. The accuracy of the procedure is seen from the fact that the survival probability around  $p_{\rm T}$  = 15 GeV for  $c_{\rm s}^2 = 1/3$  is identical for the longitudinal and the transverse expansion of the plasma. This is a direct reflection of the identity of the corresponding contours near  $r = 0$  (Fig. 1).

The  $J/\psi$  suppression at LHC energies, as indicated, becomes sensitive to the transverse flow, the shape of the survival probability changes and the largest  $p<sub>T</sub>$  for which the formation is definitely possible is enhanced by about 10 GeV (Fig. 6).

The  $\gamma$  suppression, which in our prescription is possible only at the LHC energy, is seen to be clearly affected by the speed of sound but not by the transverse expansion of the plasma (Fig. 7).

#### **4.3 Consequences of chain decays of quarkonia**

The entire discussion so far has been in terms of the directly produced  $J/\psi$ 's and T's. However, it is well established that only about 58% of the observed  $J/\psi$  in pp collisions originate directly, while 30% of them come from  $\chi_c$  decay and 12% from the decay of  $\psi'$ . Thus the survival probability of the  $J/\psi$  in the QGP can be written as

$$
S = 0.58S_{\psi} + 0.3S_{\chi_c} + 0.12S_{\psi'},\tag{34}
$$



**Fig. 6.** The survival probability for  $J/\psi$  at LHC energy with longitudinal and transverse cooling when the speed of sound is changed



Fig. 7. The survival probability for  $\Upsilon$  at (RHIC and) LHC energywith longitudinal and transverse cooling when the speed of sound is changed

in an obvious notation. We give the survival probabilities of these resonances for a transversely expanding plasma at RHIC with the speed of sound as  $1/\overline{3}^{1/2}$  in the left panel of Fig. 8. We see that the competition of the formation times and the duration of the sufficient dissociation energies render a rich detail to the suppression pattern of the charmonia. We shall later argue that the  $\psi$ 's are easily destroyed by a moderately hot hadronic gas as well, as their binding energy is on the order of just 50 MeV. The right panel of the figure shows the survival probability as a function of the transverse momentum when the speed of sound in the plasma is varied from  $1/3^{1/2}$  to  $1/5^{1/2}$ . We see that while the gross features for the shape of the survival probability remain similar to Fig. 5, as seen earlier, the survival of the  $J/\psi$  for larger  $p_T$  is now enhanced as the  $\chi_c$ 's, which decay to form  $J/\psi$ , start escaping. Thus the complete escape of



**Fig. 8.** The survival probability for directly produced  $J/\psi$ ,  $\chi_c$ and  $\psi'$  at RHIC energies, for a transversely expanding plasma (left panel), for  $c_s^2 = 1/3$ . The right panel shows the results when the decays of the resonances is accounted for



Fig. 9. Same as Fig. 8, for the survival probability of charmonia at LHC energies

the  $\chi_c$  having  $p_T > 10 \,\text{GeV}$  (for  $c_s^2 = 1/3$ ) yields a distinct kink in  $S(p_T)$  at  $p_T \approx 10$  GeV. Its location, which shifts to about 14 GeV when the speed of sound is decreased, can perhaps be more accurately determined by plotting the derivative  $dS/dp<sub>T</sub>$ , which will have a discontinuity there. If the statistics is really good (which unfortunately is a somewhat difficult proposition), this discontinuity in the  $dS/dp<sub>T</sub>$  for the  $J/\psi$  can be a unique signature of melting of the resonance in the plasma, as the  $p_T$  dependence of the survival probability due to absorption by hadrons should be weak [19] and smooth [5].

The corresponding results for the LHC energies are given in Fig. 9, in an analogous manner, and we see that the characteristic "kink" in  $S(p_T)$  has shifted to about  $p_T = 16 \,\text{GeV}$ , announcing the complete escape of  $\chi_c$  from the plasma.

The decay contribution of the resonances completely alters the shape of the survival probabilities for  $\Upsilon$  from that seen earlier (Fig. 7).

We note that (Fig. 10) both  $\chi_b$  and  $\gamma$ <sup>r</sup> get suppressed at the RHIC energies, while the directly produced  $\gamma$  is likely to escape. However, as only about  $54\%$  of the  $\gamma$ 's may be directly produced, while about 32% have their



**Fig. 10.** The survival probability for directly produced  $\gamma$ ,  $\chi_c$ and  $\gamma'$  at RHIC energies, for a transversely expanding plasma (left panel), for  $c_s^2 = 1/3$ . The right panel shows the results when the decays of the resonances is accounted for



Fig. 11. Same as Fig. 10, for the survival probability of bottomonia at LHC energies

origin in the decay of  $\chi_b$  and a further 14% in the decay of  $\Upsilon'$ , the resultant (right panel, Fig. 10) survival probability is just about 50% for the lowest  $p<sub>T</sub>$ , signalling the suppressions of the higher resonances.

The results for the LHC energies become quite dramatic (Fig. 11) as now the "kink" in the survival probability is very clearly seen at  $p_T \sim 20 \,\text{GeV}$  for  $c_s^2 = 1/3$  and at ∼ 26 GeV for the lower speed of sound. If one looks at  $dS/dp<sub>T</sub>$ , then it could be very useful indeed. We may add that fluctuations in the initial conditions et cetera may perhaps make it difficult to notice this aspect for the  $J/\psi$ 's, though for the T's these should survive, provided we have good statistics at these  $p_T$ .

The large difference in this behavior seen between charmonium and bottomonium suppression has its origin in the large difference in the energy densities required to melt the different resonances of charmonia and bottomonia.

#### **4.4 Absorption by nucleons and comovers**

So far we have discussed the fate of quarkonia only when the presence of a quark–gluon plasma is considered. It is very well established that there are several aspects like initial state scattering of the partons, shadowing of partons, absorption of the pre-resonances  $(|Q\overline{Q}q\rangle)$  states) by the nucleons before they evolve into physical quarkonia, and also dissociation of the resonances by the comoving hadrons [5,6]. It has been argued that the absorption by comoving hadrons will be important for  $\psi'$ , due to its very small binding energy, while for more tightly bound resonances it may be weak [5].

Let us briefly comment on them one-by-one. Shadowing of partons should play an important role in the reduced production of quarkonia, especially at the LHC energies. It is clear that if shadowing is important, we shall witness a larger effect on  $J/\psi$  than on  $\Upsilon$ , because of the smaller values of the x for gluons. At the same time, the effect of shadowing should be similar for different resonances of the charmonium (or bottomonium), as similar  $x$  values would be involved for them.

The absorption of the pre-resonances by the nucleons is another source of  $p_T$  dependence. It is important to recall once again that as the absorption is operating on the pre-resonance, the effect should be identical for all the states of the quarkonium which are formed.

This is a very important consideration as it is clear that if we look at the ratio of rates for different states of  $J/\psi$ or the  $\Upsilon$  family as a function of  $p_{\text{T}}$ , then in the absence of QGP effects they would be identical to what one would have expected in the absence of nuclear absorption and shadowing, providing a clear pedestal for the observation of QGP [18].

There is another aspect of the  $p_T$  dependence which needs to be commented upon, before we conclude. The (initial state) scattering of partons, before the gluons of the projectile and the target nucleons fuse to produce the  $Q\overline{Q}$  pair, leads to an increase of the  $\langle p_{\rm T}^2 \rangle$  of the resonance which emerges from the collision [20]. The increase in the  $\langle p_{\rm T}^2 \rangle$ , compared to that for pp collisions, is directly related to the number of collisions the nucleons are likely to undergo, before the gluonic fusion takes place. This leads to a clear possibility of relating the average transverse momentum of the quarkonium to the transverse energy deposited in the collision (which decides the number of participants and hence the number of collisions). Considering that collisions with large  $E_T$  may have formation of QGP in the dense part of the overlapping region, the quarkonia, which are produced in the densest part (and hence contributing the largest increase in the transverse momentum) are also most likely to melt and disappear. This may lead to a characteristic saturation and even a turn-over of the  $\langle p_{\rm T}^2 \rangle$ when plotted against  $E_T$  when the QGP formation takes place. In the absence of QGP, this curve would continue to rise with  $E_T$ .

Obviously, all these (well explored and yet non-QGP) effects need to be accounted for, before we can begin to see the suppression of the quarkonium due to the formation of QGP. It seems that this has been achieved at least at the SPS energies [5, 6].

The next step would obviously be the one discussed in the present work – that of looking for the  $p_T$  dependence of the survival probability, to see if we can get more detailed

information on the equation of the state of the plasma. This would require high precision data, extending to larger  $p_T$  at several  $E_T$ . This may prove to be difficult, though not impossible in principle at least. It will, however, prove to be very valuable, if it can be done.

# **5 Summary and discussion**

We have seen that the survival probability of  $J/\psi$  at RHIC energies and that of  $\Upsilon$  at LHC energies can provide valuable information about the equation of state of the quark matter, as the results are not affected by uncertainties of the transverse expansion of the plasma. If the transverse expansion of the plasma takes place, it gives a distinct shape to the survival probability of the  $J/\psi$  at LHC energies, whose detection will be a sure signature of the transverse flowof the plasma within the QGP phase.

Before concluding we would add that in these exploratory demonstrations we have chosen some specific values [8] for the deconfining matter which can dissociate quarkonia. These could be different; in particular the  $\epsilon_s^{J/\psi}$ could be much larger than the value used here. This, however, will not change the basic results as the time scales involved in the  $\gamma$  and the  $J/\psi$  suppression are so very different.

The other uncertainty comes from the usage of  $\epsilon_s$  as the criterion for deconfinement. One could have as well used the Debye mass as the temperature and the fugacity changed in a chemically equilibrating plasma, to fix the deconfining zone. This can indeed be done, along with the other extremum of the Debye mass estimated from lattice QCD. This has been studied in great detail by the authors of [18] and can be easily extended to the present case. We plan to do this in a future publication.

In brief, we have shown that the  $J/\psi$  and  $\Upsilon$  suppression at RHIC and LHC energies can be successfully used to map the equation of state for the quark matter. As the two processes will map different but overlapping regions, taken together, these results will help us to explore a vast region of the equation of state.

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